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Vacuum stress around a topological defect

V. A. De Lorenci* and E. S. Moreira Jr.†
Instituto de Ciências, Universidade Federal de Itajubá,
Av. BPS 1303 Pinheirinho, 37500-903 Itajubá, MG, Brazil
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We show that a dispiration (a disclination plus a screw dislocation) polarizes the vacuum of a scalar field giving rise to an energy momentum tensor which, as seen from a local inertial frame, presents non vanishing off-diagonal components. Such a new effect resembles that where an induced vacuum current arises around a needle solenoid carrying a magnetic flux (the Aharonov-Bohm effect). The results may have applications in cosmology (chiral cosmic strings) and condensed matter physics (materials with linear defects).

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It is fairly well known that a needle solenoid carrying a magnetic flux makes virtual charged particles to run around the solenoid inducing a non vanishing current density (see e.g. Ref [1]). We wish to consider what seems to be a gravitational (geometric) analogue of this Aharonov-Bohm effect, by computing the vacuum expectation value of the energy momentum tensor of a massless and neutral scalar field far away from a dispiration.

Let us begin by presenting the geometry of the background (units are such that $c = \hbar = 1$),

$$ds^2 = dt^2 - dr^2 - \alpha^2 r^2 d\theta^2 - (dz + \kappa d\theta)^2, \tag{1}$$

where the points labeled by (t, r, θ, z) and $(t, r, \theta + 2\pi, z)$ are identified [2, 3]. When $\alpha = 1$ and $\kappa = 0$ Eq (1) becomes the line element of the flat spacetime written in cylindrical coordinates. Borrowing terminologies in condensed matter physics, the parameters α and κ correspond to a disclination and a screw dislocation, respectively. We should remark that Eq (1) may be associated with the gravitational background of certain chiral cosmic strings [4] (as has been suggested in Ref. [2]), as well as can describe (in the continuum limit) the effective geometry around a dispiration in an elastic solid (see Ref. [5] and references therein).

The definitions $\varphi := \alpha\theta$ and $Z := z + \kappa\theta$ lead to

$$ds^2 = dt^2 - dr^2 - r^2 d\varphi^2 - dZ^2, \tag{2}$$

which should be considered together with the peculiar identification

$$(t, r, \varphi, Z) \sim (t, r, \varphi + 2\pi\alpha, Z + 2\pi\kappa). \tag{3}$$

Although Eq. (2) expresses the fact that the background is locally flat, due to Eq. (3) we cannot use Eq. (2) (which is a local statement) to infer that the global symmetries of the background are the same as those of the Minkowski spacetime (in this sense Eq. (2) is singular). In fact, Eq. (2) disguises a curvature singularity on the symmetry axis [2] (when $\kappa \neq 0$, in the context of the Einstein-Cartan theory, there is also a torsion singularity at $r = 0$ [3, 6]).

As is well known (see e.g. Ref. [7]) the vacuum expectation value of the energy momentum tensor can formally be obtained by applying the differential operator

$$\mathcal{D}^\mu{}_\nu(x, x') := (1 - 2\xi)\nabla^\mu\nabla_{\nu'} - 2\xi\nabla^\mu\nabla_\nu + (2\xi - 1/2)\delta^\mu{}_\nu\nabla^\lambda\nabla_{\lambda'} \tag{4}$$

to the renormalized scalar propagator around a dispiration,

$$\langle T^\mu{}_\nu\rangle = i \lim_{x' \to x} \mathcal{D}^\mu{}_\nu(x, x')\ D^{(\alpha,\kappa)}(x, x'). \tag{5}$$

We have recently obtained $D^{(\alpha,\kappa)}(x, x')$ (classical propagators have been considered in Ref [8]) by using the Schwinger proper time prescription combined with the completeness relation of the eigenfunctions of $\Box$ [9]. Such eigenfunctions have the form $R(r)\chi(\varphi)\exp\{i(\nu Z - \omega t)\}$ which, by observing Eq. (3), leads to

$$\chi(\varphi + 2\pi\alpha) = e^{-i2\pi\nu\kappa}\chi(\varphi). \tag{6}$$

This boundary condition is typical of the Aharonov-Bohm set up where $\nu\kappa$ is identified with the flux parameter $e\Phi/2\pi$. If we carry over to the four-dimensional context lessons from gravity in three dimensions [10, 11], it follows that the charge e and the magnetic flux Φ should be identified with the longitudinal linear momentum ν and $2\pi\kappa$, respectively [2].

The renormalized propagator is given by

$$D^{(\alpha,\kappa)}(x,x') = \frac{i}{2\pi^2}\sum_{n=-\infty}^{\infty}\int_0^{\infty} d\tau \frac{\left[\tau^2+\pi^2-(2\pi\alpha n-\Delta\varphi)^2\right]\left[r^2+r'^2+2rr'\cosh\tau+(\Delta Z-2\pi n\kappa)^2-(\Delta t)^2\right]^{-1}}{\{[\pi(2\alpha n+1)-\Delta\varphi]^2+\tau^2\}\{[\pi(2\alpha n-1)-\Delta\varphi]^2+\tau^2\}}, \quad (7)$$

where $\Delta t := t - t'$, likewise for φ and Z [9]. As $\kappa \to 0$ the dominant contribution in Eq. (7) is the renormalized scalar propagator in an ordinary conical background [12]. Therefore when $\kappa/r \to 0$, Eq. (5) yields for the diagonal components essentially the expressions long known in the literature for the vacuum fluctuations around an ordinary cosmic string ($\kappa = 0$) [13]. Regarding the remaining components, the prescription in Eq. (5) kills off the dominant contribution in Eq. (7), resulting that the subleading contribution yields two non vanishing off-diagonal components,

$$\langle T^{\varphi}{}_{Z}\rangle = \frac{i}{r^2}\lim_{x'\to x}\partial_{\varphi}\partial_{Z}D^{(\alpha,\kappa)}(x,x') = \frac{\kappa}{r^6}B(\alpha), \quad (8)$$

and

$$\left\langle T^{Z}{}_{\varphi}\right\rangle = \frac{\kappa}{r^4}B(\alpha), \quad (9)$$

where

$$B(\alpha) := \frac{1}{32\pi^3\alpha^2}\int_0^{\infty} d\tau \frac{\alpha\sin(\pi/\alpha)\left[\cos(\pi/\alpha)-\cosh(\tau)+\tau\sinh(\tau)\right]-\pi\left[\cos(\pi/\alpha)\cosh(\tau)-1\right]}{\left[\cosh(\tau)-\cos(\pi/\alpha)\right]^2\cosh^4(\alpha\tau/2)}. \quad (10)$$

It is worth remarking that, unlike the diagonal components, $\langle T^{\varphi}{}_{Z}\rangle$ and $\left\langle T^{Z}{}_{\varphi}\right\rangle$ do not depend on the coupling parameter ξ.

The plot of $B(\alpha)$ against the disclination parameter α is shown in Fig. 1. When $\alpha = 1$, the integration in Eq. (10) can be analytically evaluated [14], resulting $B = 1/60\pi^2$ which corresponds approximately to the value of α suggested by the physics of formation of ordinary cosmic strings [13].

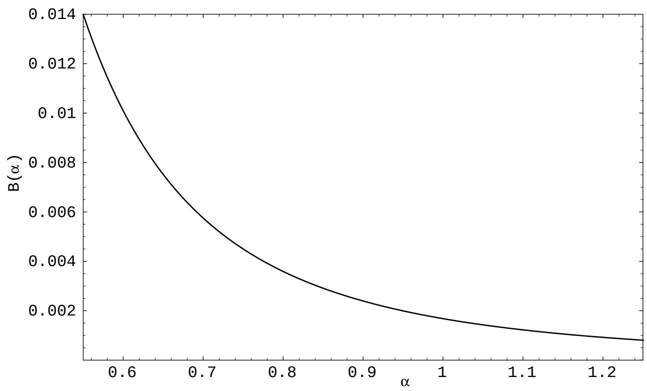

FIG. 1: Plot $B(\alpha)$ versus α.

It is instructive to display both disclination and screw dislocation effects in a same array. When $\xi = 1/6$ (conformal coupling), for example, $\langle T^{\mu}{}_{\nu}\rangle$ with respect to the local inertial frame [cf. Eq. (2)] can be cast into the form

$$\langle T^{\mu}{}_{\nu}\rangle = \frac{1}{r^4}\begin{pmatrix} -A & 0 & 0 & 0 \\ 0 & -A & 0 & 0 \\ 0 & 0 & 3A & \kappa B/r^2 \\ 0 & 0 & \kappa B & -A \end{pmatrix}, \quad (11)$$

where $A(\alpha) := (\alpha^{-4}-1)/1440\pi^2$, and which holds far away from the defect (and for $\alpha \neq 1$, when $\kappa \neq 0$). [When $\kappa \neq 0$, by setting $\alpha = 1$ in Eq. (11), A vanishes and subleading contributions depending on κ take over.]

Before closing this note, let us interpret the polarization effect displayed in Eq. (9) in the light of the analogy with the Aharonov-Bohm effect following Eq. (6). Observing Eq. (9), we can say that a dispiration (more precisely, a screw dislocation) polarizes the vacuum of a scalar field, inducing a flux of longitudinal linear momentum around the defect. Such a flux depends on the direction of the screw dislocation (i.e., on the sign of κ) in the same way that vacuum currents around a needle solenoid depend on the direction of the magnetic flux.

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* Electronic address: delorenci@unifei.edu.br
† Electronic address: moreira@unifei.edu.br

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